Doppler Radar Formulas
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Distance, or "range" to a RADAR echo is given by the formula

$$R = \frac{cT}{2}$$

where   R=range (distance to echo)
            c = speed of electromagnetic radiation = 3×10^8 m s^{-1}
            T = time since pulse was emitted

NEXRAD assumes that any echo it is "seeing" is generated by the most recent pulse that was emitted. However, a pulse can be sent out and bounce off of a distant target, then a second pulse may be sent out before the first pulse has made it back to the radar. This creates an ambiguity where the radar cannot tell whether an echo is from the most recent pulse reflecting from a target closer to the radar, or a previous pulse reflecting from a target more distant from the radar. The distance that a pulse can travel out and back before the next pulse is emitted defines this limitation and is called the "maximum unambiguous range" (R_{max}). Clearly, R_{max} will be limited by the time between the pulses—the shorter the time, the shorter the distance traveled before the next pulse. Therefore, the greater the frequency (more pulses per second), the shorter is the time between the pulses, and the shorter is the maximum unambiguous range.

Maximum unambiguous range:

$$R_{max} = \frac{c}{2PRF}$$

where R_{max} = maximum unambiguous range

$c = 3 \times 10^8 \text{ m s}^{-1}$

PRF = pulse repetition frequency [s^{-1}], i.e., number of pulses per second

Example—calculating R_{max}:

If the NEXRAD is operating at a PRF of 326.09 s^{-1}, what is the maximum unambiguous range?

$$R_{max} = \frac{c}{2PRF} = \frac{300{,}000{,}000 \text{ m s}^{-1}}{2 \cdot 326.09 \text{ s}^{-1}} = 459996 \text{ m} = 460 \text{ km}$$

$$R_{max} = 459996 \text{ m} = 460 \text{ km} \times \frac{0.539612 \text{ nm}}{1 \text{ km}} = 248.2 \text{ nm}$$

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Fundamental sampling theorem: To measure a frequency, f_d, it is necessary to sample at a frequency of at least $2f_d$.

The sampling rate is the PRF, so

$$2f_d = PRF$$

Doppler theorem:

$$V = \frac{f_d \cdot \lambda}{2}$$

where f_d = Doppler frequency shift

λ = wavelength

V = radial velocity, i.e. component of velocity toward of away from the radar

Maximum unambiguous Doppler velocity, v_{max}, is

$$V_{max} = \frac{PRF \cdot \lambda}{4}$$

Example—calculating V$_{max}$:

If the NEXRAD is operating at a PRF of 326.09 s^{-1}, what is the maximum unambiguous velocity?

First, convert NEXRAD wavelength from centimeters to meters:

$$\lambda = 10 \; cm \times \frac{1 \; m}{100 \; cm} = 0.10 \; m$$

$$V_{max} = \frac{PRF \cdot \lambda}{4} = \frac{326.09 \; s^{-1} \cdot 0.10 \; m}{4} = 8.152 \; ms^{-1}$$

$$V_{max} = 8.152 \ ms^{-1} \times \frac{1.94384449 \ kt}{1 \ ms^{-1}} = 15.8 \ kt$$

The "Doppler Dilemma:" Since the maximum unambiguous range is *inversely* related to the PRF while the maximum unambiguous Doppler velocity is *directly* related to the PRF, *there is no single PRF that can maximize both at the same time.*

Beam Height

The radar beam does not travel in a straight line in the earth's atmosphere. Under standard atmospheric conditions, it undergoes "standard refraction" or bending following the earth's surface:
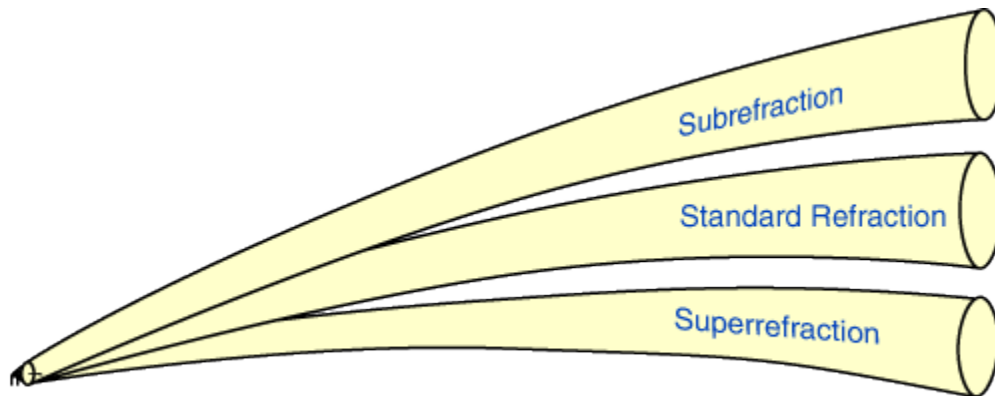
The bending of the beam is still not as great as the curvature of the earth, so the beam increases in elevation as range from the radar increases. The beam height above sea level or radar level

can be calculated for a standard atmosphere using the following formula (Rinehart 2010):

$$H = \sqrt{R^2 + \left(\frac{4}{3}r_e + H_0\right)^2 + 2R\left(\frac{4}{3}r_e + H_0\right)\sin\varphi} - \frac{4}{3}r_e$$

H = height of the beam centerline above sea level in km, or above radar level if H_0 is set to 0.

H_0 = height of the radar above sea level = station elevation + tower height

R = range (distance) to the targets in km.

φ = elevation angle of beam in degrees

r_e = radius of earth, 6374 km

For product generation, the metric units (km) are converted to nm and kft as necessary."

Example—calculating beam height, H:

For an elevation angle of 0.5 degrees, what is the beam height above radar level at a distance (range) of 90 nm from the Melbourne, Florida, NEXRAD? What is the beam height above sea level?

First convert the distance to km:

$$R = 90\ nm \times \frac{1.852\ km}{1\ nm} = 166.68\ km$$

$$H = \sqrt{R^2 + \left(\frac{4}{3}r_e + H_0\right)^2 + 2R\left(\frac{4}{3}r_e + H_0\right)\sin\varphi} - \frac{4}{3}r_e$$

For ARL (above radar level), $H_0 = 0$ m $= 0$ km.

$$H =$$

$$= \sqrt{(166.68)^2 + \left(\frac{4}{3}\cdot 6374 + 0\right)^2 + 2R\left(\frac{4}{3}\cdot 6374 + 0\right)\sin(0.5)}$$
$$- \frac{4}{3}\cdot 6374$$

$$H = 3.088482517\ km \times \frac{3280.8\ ft}{1\ km} = \mathbf{10133\ ft\ ARL}$$

For the beam height above sea level, add in the elevation and tower height of the MLB radar:

Elevation $= 35$ ft $= 10.7$ m.

Tower height $= 20$ m.

$H_0 = 10.7$ m $+ 20$ m $= 30.7$ m $= 0.0307$ km.

$$H =$$

$$= \sqrt{(166.68)^2 + \left(\frac{4}{3}\cdot 6374 + 0.0307\right)^2 + 2R\left(\frac{4}{3}\cdot 6374 + 0.0307\right)\sin(0.5)}$$
$$- \frac{4}{3}\cdot 6374$$

$$H = 3.119176617km \times \frac{3280.8\ ft}{1\ km} = \mathbf{10234\ ft\ MSL}$$