Derive the Rain Rate R (mm/hr) from the Marshall-Palmer z-R Relationship

$$z = 200R^{1.6}$$

Take the log of both sides of the z-R relationship:

$$\log z = \log 200 + \log R^{1.6}$$

Apply rules of logs and exponents:

$$\log z = \log 200 + 1.6 \log R$$

Subtract log 200 from both sides:

$$\log z - \log 200 = 1.6 \log R$$

Divide both sides by 1.6:

$$\frac{\log z - \log 200}{1.6} = \log R$$

Get log R on the left-hand side:

$$\log R = \frac{\log z}{1.6} - \frac{\log 200}{1.6}$$

To get rid of the log function on R, raise both sides as a power of 10; since the "log of R" is just the power that you need to raise 10 to, to get R, raising 10 to a power of log R gives R:

$$R = 10^{\log R} = 10^{\left(\frac{\log z}{1.6} - \frac{\log 200}{1.6}\right)}$$

Apply rules of exponents:

$$R = 10^{\left(\frac{\log z}{1.6}\right)} \cdot 10^{-\left(\frac{\log 200}{1.6}\right)}$$

The factor on the right is just a numerical value:

$$R = 10^{\left(\frac{\log z}{1.6}\right)} \cdot 0.036463324$$

Rearrange:

$$R = 0.036 \cdot 10^{\left(\frac{\log z}{1.6}\right)}$$

$$= 0.036 \cdot 10^{0.625 \cdot \log z}$$

But the reflectivity, Z, in decibels of reflectivity, is just 10 times the log of the radar reflectivity factor, z:

$$dBZ = 10 \cdot \log z \Longrightarrow \frac{dBZ}{10} = \log z$$

Substitute this expression for log z into the equation:

$$R = 0.036 \cdot 10^{0.625 \cdot \frac{dBZ}{10}}$$

The final result is this expression that can be used to calculate the rain rate in mm/hr from the reflectivity obtained from the radar:

$$\boxed{R = 0.036 \cdot 10^{0.0625 \cdot dBZ}}$$