Notes on Logarithms

Logarithms are just exponents. Consider a constant, "a" and two variables x and y. If "a" raised to the power of y is equal to x, then we say that y is the logarithm to the base "a" of x:

$$a^y = x \Rightarrow y = \log_a x$$

In other words, the logarithm or log of x is the power that "a" must be raised to, to equal x. For the so-called common log, the base is just 10, and often it is implied rather than written explicitly:

$y = \log_{10} x$ is written as $y = \log x$

The "natural log" uses the number "e" as the base where e = 2.718281828459… and usually is written as ln:

$$y = \log_e x = \ln x$$

We will use the ln x notation. The exponential function $e^y = x$ is the inverse function of $y = \ln x$.

Logarithms have the useful property of transforming multiplication operations to addition and division to subtraction. Here we list some properties of logarithms:

$$\ln(M \cdot N) = \ln(M) + \ln(N) \neq \ln(M + N)$$

$$\ln\left(\frac{M}{N}\right) = \ln(M) - \ln(N) \neq \ln(M - N)$$

$$\ln(M^P) = p \cdot \ln(M)$$

$$e^{\ln x} = x$$

$$\ln(e^x) = x$$

Proof of the last property:

$$\ln(e^x) = x \cdot \underbrace{\underbrace{\ln e}_{e^1 = e}}_{1 = \ln e} = x$$