

Sample Calculations Wx365

Stefan-Boltzmann Law

$$E^* = \sigma T^4$$

If one object is twice as hot as another object, how many times more energy does it emit?

$$E_{\text{cool}}^* = \sigma T_{\text{cool object}}^4$$

$$E_{\text{warm}}^* = \sigma T_{\text{warm object}}^4$$

$$T_{\text{warm object}} = 2 \cdot T_{\text{cool object}}$$

$$\begin{aligned} E_{\text{warm}}^* &= \sigma T_{\text{warm object}}^4 = \sigma (2 \cdot T_{\text{cool object}})^4 \\ &= \sigma \cdot 2^4 \cdot T_{\text{cool object}}^4 = \sigma \cdot 16 \cdot T_{\text{cool object}}^4 \end{aligned}$$

$$E_{\text{warm}}^* = 16 (\sigma T_{\text{cool object}}^4) = 16 \cdot E_{\text{cool object}}^*$$

→ 16 times = 2^4 times more energy

How many times more energy (per unit area)
does the sun emit than the earth?

$$T_{\text{sun}} \approx 6000 \text{ K}$$

$$T_{\text{earth}} \approx 300 \text{ K}$$

$$300 \text{ K} \overline{) 6000 \text{ K}}^{20}$$

Sun is 20 times hotter than earth.

Sun emits 20^4 times more energy!

$20^4 = \underline{160,000}$ times more per
square meter of sfc area!!

Wien's Law

$$\lambda_{\max} = \frac{2897 \mu\text{m} \cdot \text{K}}{T}$$

Remember, if given a temperature in $^{\circ}\text{C}$, must convert to K before using this equation!

$$\text{Say } T = 20^{\circ}\text{C} \longrightarrow T \cong 20^{\circ}\text{C} + 273\text{K} \\ \cong 293\text{K}$$

$$\lambda_{\max} = \frac{2897 \mu\text{m} \cdot \text{K}}{293\text{K}} = \boxed{9.88 \mu\text{m}}$$

$$\lambda_{\max} = \frac{2897 \mu\text{m} \cdot \text{K}}{T} \Rightarrow T \cdot \lambda_{\max} = 2897 \mu\text{m} \cdot \text{K}$$

$$\Rightarrow T = \frac{2897 \mu\text{m} \cdot \text{K}}{\lambda_{\max}}$$

Let's say we measure the maximum amount of energy coming from the sun at 0.5082 μm . What is the temperature of the sun?

$$T = \frac{2897 \cancel{\mu\text{m}} \cdot \text{K}}{0.5082 \cancel{\mu\text{m}}} = \boxed{5700\text{K}}$$

Greenhouse Effect

$$T_{\text{sun}} \approx 6000 \text{ K}$$

$$\lambda_{\text{max(sun)}} = \frac{2897 \mu\text{mK}}{T_{\text{sun}}} \approx \frac{3000 \mu\text{mK}}{6000 \text{ K}}$$

$$\approx \boxed{0.5 \mu\text{m}}$$

$$T_{\text{earth}} \approx 300 \text{ K}$$

$$\lambda_{\text{max(earth)}} = \frac{2897 \mu\text{mK}}{T_{\text{earth}}} \approx \frac{3000 \mu\text{mK}}{300 \text{ K}}$$

$$\approx \boxed{10 \mu\text{m}}$$